

# **Bucket Investing**



# With Dynamic Risk-Managed Portfolios

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### **Bucket Investing Overview**

Designing an asset allocation approach to meet a client's financial objectives often is cited as the most important component of the investment advisory process. Despite the advent of Markowitz and Modern Portfolio Theory, this process cannot be easily reduced to a financial engineering problem. Economists have long made a key assumption within their models that is highly flawed for the purposes of simplifying the mathematical implementation: they assume that human beings only make rational decisions.

Nobel Prize winner Daniel Kahneman, a renowned expert in the field of "behavioral finance," has said that: "Economists think of what people ought to do, Psychologists watch what they actually do." Statman (2005) suggested that investors are neither irrational nor rational but rather "normal"—that is, they display elements of both characteristics.

Empirical evidence has shown that investors are indeed far from rational. In a classic study by Dalbar, the average equity mutual fund investor made a return of 5.02%, while the S&P 500 made 9.22% over the same 20-year period. The 4.20% return differential could not be explained by financial theory, and was famously dubbed "the behavior gap." Investors lost over 60% of the returns available to them through poor market timing that was driven by emotional decision-making.

It can be argued that the most important job for an investment advisor or financial planner is simply to protect investors from themselves. Creating an asset allocation approach that can keep them invested and avoid selling risky investments at the wrong time is a difficult challenge. The "bucket" approach to investing has emerged as a popular asset allocation methodology in the financial planning and advisory community because it is specifically designed to account for actual investor behavior (Benjamin, 2011). Furthermore, it is highly compatible with the traditional financial planning objectives which require matching assets to meet future liabilities.

The essence of the bucket approach is to divide a client's portfolio assets into several pools, or "buckets," each with different planned goals, needs, or time horizons, and then design a separate asset allocation policy for each "bucket." For different investors, an individualized bucketing approach also reflects financial planners' and advisors' emphasis on case-by-case tailored solutions for their fee-paying clients.



Asset allocation using the bucket approach utilizes discrete "buckets" assigned to asset type, such as bonds for income and capital preservation, or equities/stocks for capital appreciation. The simplest implementation of bucket investing would use only two buckets: bonds or cash to meet short-term expenses, and stocks for long-term growth (Evensky and Katz, 2006).



There are also more complicated "bucketing" strategies that use three to six buckets (Beaudoin, 2013). Intermediate "buffer buckets" can have more refined planning time horizons designed for growth or spending goals and thus be targeted at layered return objectives. From an investment management standpoint, a more complex bucket approach could use different portfolios for each bucket, and these portfolios could be formed using either assets or strategies (or both) with either a passive or active management overlay.

Throughout this paper we utilize a "wasting" bucket approach whereby we drain the income bucket of its assets via a fixed percentage annual withdrawal for ten years before turning to the stock bucket to generate the requisite returns to support the ability to make the annual withdrawal. Many financial advisors instead use a "waterfall" bucket approach (as is illustrated in the 3-Bucket Retirement Income graphic).

In the "waterfall" bucket approach, the income bucket is replenished yearly with flows from the longer-term bucket. This takes time and discipline to accomplish especially if done more aggressively, for example, guarterly. These replenishments occur whether or not these longer-term buckets have generated sufficient returns to fund the withdrawal needed each year. In this way in a bear market they can eat into "principal" but the result is a more consistent balancing of risk among all of the buckets in total. However, as some commentators have noted it can be seen as an inefficient use of capital in that it overly invests in liquid assets during rising stock markets. As the examples in the Appendix point out, this approach has further advantages as the number of buckets used increases.

There are many reasons why a distinct bucketing strategy design might be appropriate for different investors, such as different tax brackets and/or opportunities to shelter income from taxation, or different planning objectives. For example, some investors have relatively shortterm objectives, while others may have longer-term goals like saving for college tuition payments, retirement spending, or estate planning. To address these specific needs within the context of a client's unique situation, the "bucket" approach often works well in a planning or advisory practice (Lucia 2004 & 2010).

In addition, part of the reason the bucket approach has gained popularity in the investment advisory and planning community is the anecdotal evidence from peers that it improves client communication and retention. Behavioral finance proponents attribute this success mainly to the fact that most investors have a "mental accounting" bias.

Coined first by academic researcher Richard Thaler (1999), "mental accounting" describes a person's tendency to categorize and evaluate economic outcomes by grouping their assets into a number of non-fungible (non-interchangeable) "mental accounts." People may alter their perspective on money and investment according to the surrounding circumstances and make irrational decisions due to such a framing bias. Behavioral life cycle theory (Shefrin and Thaler, 1988) submits that people mentally allocate wealth across three classifications: current income, current assets, and future income. The propensity to spend is greatest from the current income account, while many treat the source of future income differently.

A time-horizon-based "bucketing" approach for wealth management was designed to address psychologically both the safety of near-term liquidity need and the goal of long-term growth of wealth. In practice, a floor level of assets designated as a short-term "spending bucket" is often kept as cash or in short-term securities that have little or no investment risk. Further, from a portfolio management perspective, planning "buckets" of capital under the framework of "goals-based investing" (Nevins, 2004), does institute beneficial risk discipline into the investment process.

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Richard Thaler (1999)

# The Myth of Time Diversification

One of the most important assumptions underlying the bucket approach is the theory of "time diversification." This refers to the concept that investments in riskv assets such as stocks are actually less risky over longer periods than shorter ones. In a traditional two-bucket approach, the least risky asset-typically short-term bonds—is held in the first bucket and used to service income needs, while the risky portion of the portfolio is invested in stocks (for the second bucket) for sufficient time to overcome any "bad luck" in terms of the start date (i.e. beginning of a bear versus the start of a bull market).

This assertion that sufficient time will reduce the riskiness of stocks is the subject of much debate in the academic community (see Kritzman, "What Practitioners Need to Know ... About Time Diversification," Siegel, "Stocks for the Long-Run," and Samuelson, "The Long-Term Case for Equities—and how it can be oversold"). However, the numbers and the math are fairly straightforward and suggest that this matter is much more settled than in question.

Kritzman notes that the key point of confusion is that the probability of losing money-which is mathematically and empirically supported to be lower over time-does not consider the magnitude of the potential losses. Like any investment, the "expectation" is a function of both the probability of winning or losing and the ratio of the size of profits to losses. The dispersion of compound returns (percentages) does shrink over time, but the dispersion of ending portfolio wealth (terminal wealth-the dollar value) actually increases over time. All of this is captured in financial options theory and pricing—and, in fact, the cost of option premiums does increase over time, which



Figure 1



reflects the truth that time diversification is illusory (Bodie 1995). Failure to understand this can lead to some faulty construction of the longer-term buckets.

To illustrate this concept we performed a series of Monte Carlo simulations using Brownian motion, which is a typically accepted practice for modelling financial time series. Above is the distribution of compound returns for 5-year simulations assuming that the market has a return of 8% and a standard deviation of 20%.

We performed 25,000 simulations and recorded the result (Figure 1).

Notice that there is considerable variability when holding stocks for such a short time period. While the distribution is skewed, maximum and minimum values are nearly identical.

Subsequently, we performed a simulation using a long time period instead. In this case, to make the results clear, we chose 100 years (Figure 2). Clearly, it is easy to see that as more time passes the distribution shifts to be almost entirely positive at every point in the distribution. These results are exactly the same as the ones produced by other authors to support the notion of time diversification.

The problem is that investors do not just get 100-year returns. Instead, they must compound them over time—in other words, every year they must effectively re-invest their portfolio and have their entire wealth fluctuate as a function of the next year's investment return. This is why risk management is so important, because a 50% loss can erase over ten years of investment gains.

In the next set of simulations we show the distribution of terminal wealth (ending portfolio wealth) as a function of an investor's holding period using the same set of assumptions.

Notice that the maximum terminal wealth is just over \$750,000, while the minimum is close to \$20,000. For comparison we ran the same study using a 20- and then 100-year time frame.

In both the 20- and 100-year holding periods there are multiple instances of portfolios that have an ending wealth that is below \$10,000 (i.e. a greater-than 90% loss!), and in some cases close to zero! Of course, the magnitude of the possible gains is also substantially higher than for the 5-year portfolio, in some cases with billions of dollars being earned in the 100-year portfolio. Clearly, the variation in the outcomes of terminal wealth increases substantially over time. Dispersion of Terminal Wealth for Simulated Stock Time Series





Figure 4



Distribution of Terminal Wealth 100-year

Figure 5

# The Sequence of Returns Dilemma

There are other related assumptions that are important to consider when using the bucket approach. For example, the sequence of returns is essential to determining ending portfolio wealth and whether or not an investor will be successful in achieving desired results. Milevsky ("Can Buckets Bail-Out a Poor Sequence of Investment Returns?", 2006) claims that using the bucket approach or any other type of time-based asset allocation methodology is, in fact, just an optical illusion. Using the bucket approach cannot protect an investor from a poor sequence of returns; although it can potentially shift the risk to a different point in time depending on when the investor is most exposed to equities.

The key point that Milevsky makes is that, in fact, without doing more, the asset allocation of an investor using the bucket methodology will change substantially over time. For example, in Figure 6, we perform a 20-year simulation with stocks and bonds with a 4% withdrawal rate using a two-bucket approach, with each bucket given a 10-year time horizon. Starting with a 50/50 portfolio of stocks and bonds, the bucket investor eventually shifts entirely 100% to stocks. In this simulation, the actual average allocation was 77% in stocks and 23% in bonds which is a significant departure from the original 50/50 portfolio. The actual average asset allocation and timing of the shifts will depend on the underlying market performance, the number of buckets, and the length of each bucket's time horizon.

This implies that the bucket approach is not easily comparable to a withdrawal strategy with, for example, a constantly rebalanced portfolio. However, as a general statement, the bucket approach is most exposed to equity risk at the end of the period and somewhat less exposed to risk at the beginning of the investment period. At the outset, withdrawals will be taken from the less risky asset and therefore create less dependency on equity volatility than a systematic withdrawal approach from a constantly rebalanced portfolio. This can present problems depending upon when equity or fund income risk increase and losses are incurred.

#### A Simulation of Asset Allocation over Time for the Bucket Approach

% in bonds	% in stocks
50%	50%
52%	48%
51%	49%
60%	40%
52%	48%
53%	47%
43%	57%
38%	62%
30%	70%
24%	76%
22%	78%
0%	100%
0%	100%
0%	100%
0%	100%
0%	100%
0%	100%
0%	100%
0%	100%
0%	100%
0%	100%

Figure 6<sup>1</sup>

<sup>1</sup> Table assumes an 8% CAGR and 20% standard deviation for the stock basket and a 5% CAGR and 6% standard deviation for the bond basket. Correlation was assumed to be zero and 4% was withdrawn annually. 25,000 simulations were performed.

## Case Studies: Simulations of the Bucket Approach versus Systematic Withdrawal Methods

To better illustrate some of these concepts, we ran a series of case studies using Monte Carlo simulation. We compare three basic methods: 1) Bucket Approach-this used a simple two-bucket approach (50% in bonds for the first bucket and 50% in stocks for the second bucket) where the bucket was liquidated and shifted to the next bucket at the end of its respective time horizon. We chose ten years as the time horizon for each bucket. 2) Systematic Withdrawal (SW)this assumes a constantly rebalanced portfolio fixed at 50% in stocks and 50% in bonds. 3) Systematic Withdrawal Time-Weighted (SWT)—this is designed to be somewhat more comparable to the average asset allocation of the bucket approach. We used 2/3 in stocks and 1/3 in bonds in this case to reflect the fact that the bucket method has an investment in the stock portfolio that is twice as long as the investment in bonds.

We assumed that stocks would have an 8% return with a 20% standard deviation, and bonds would have a 5% return and a 6% standard deviation. The correlation between stocks and bonds was assumed to be zero. These assumptions are fairly reasonable from a historical standpoint. For withdrawal, we assumed the investor would withdraw 4% annually for a total time period of 20 years. Using the bucket approach, each bucket lasted ten years and the first bucket contained bonds while the second contained stocks. To make accurate conclusions, we ran 25,000 different simulations and averaged the results.

One of the key metrics that we used to evaluate "success" for the investor was the Omega statistic (see Keating and Shadwick 2002: "A Universal Performance Measure"). This is used in option pricing and is a comprehensive risk measure that captures the upside versus the downside given a threshold return. The calculation of the Omega statistic is shown in Figure 7.

$$\Omega(r) = \frac{\int_r^\infty (1 - F(x)) \, dx}{\int_{-\infty}^r F(x) \, dx}$$

Figure 7

The Omega ratio is a relative measure of the likelihood of achieving a given return, such as a minimum acceptable return or a target return. Omega represents a ratio of the cumulative probability of an investment's outcome above an investor's defined return level (a threshold level), to the cumulative probability of an investment's outcome below an investor's threshold level. The Omega concept neatly captures the notion of continuous expectation on investment. It divides expected returns into two parts—upside and downside, i.e. those returns above the expected rate (the upside) and those below it (the downside). Therefore, in simple terms, consider Omega as the ratio of upside returns (good) relative to downside returns (bad). The higher the Omega value is, the greater the probability that a given return will be met or exceeded.

For the purposes of our simulations, we randomly generated 1-year returns until we had a 20-year sample. To calculate Omega, we used the annualized returns of each simulation as returns in the inputs into the function.

For a base case (Figure 8), we wanted to show that there is no difference between the bucket approach and systematic withdrawal under the assumption that one uses the same asset in each bucket. This implies that creating different time horizons or liquidation schedules has no impact on performance. In all cases "ATV" represents the average terminal value, and "MAR" represents the return relative to the maximum drawdown (mdd). Notice that the statistics are nearly identical between SW and the bucket approach. This is to be expected, since it is the same investment with the same risk and return used. In the first case, we show a more realistic example using reasonable assumptions for both stocks and bonds.

In this first case (Figure 9) we see that the systematic withdrawal approach (SW) has the highest Omega value—implying that it is the best choice for achieving a 4% withdrawal rate in the long run. The time-weighted systematic withdrawal had the next best Omega, followed by the bucket approach. The MAR shows identical reward to risk rankings as the Omega in this case.

What is interesting is that the probability of failure is, in fact, lower for the bucket approach versus the time-weighted systematic withdrawal approach. This is consistent with the literature that use probability of failure approaches to justify the use of the bucket approach. However, as we have indicated, the Omega is a more comprehensive measure to evaluate success.

Another predictable outcome was that the bucket approach had the highest average terminal value (ATV) among the different approaches. The maximum portfolio wealth was also substantially higher for the bucket approach. This reflects the fact that: 1) the bucket method has a higher average stock allocation over time; 2) withdrawals are deferred from the equity bucket for ten years which shields the compound returns from the damaging effects of withdrawing early under volatile conditions; and 3) by avoiding rebalancing between stocks and bonds, the equity allocation is allowed to compound over time and grow within the portfolio. In summary, while using the bucket approach with traditional asset classes can potentially increase the return to investors, it does so at the cost of exposing them to a potentially lower chance of financial planning success (i.e. reaching their expected annual goal).

To examine the impact of the sequence of returns, we take a look at two different circumstances, the first with bad luck at the end of the period in the form of a 50% bear market in stocks in the last year (Figure 10), and the other where the bear

#### Base Case Example: 10% return/10% risk and 5% withdrawal same asset used in bucket and SW

		max	% fail	return	risk	mdd	omega	mar
SW \$	384,554.66	\$ 2,253,507.00	0.02%	6.18%	9.78%	15.48%	88.14	0.399
Bucket \$	384,727.21	\$ 2,039,599.02	0.02%	6.18%	9.77%	15.46%	87.39	0.400

Figure 8

Case 1: Stocks and Bonds in 50/50, 8% return/20% risk for stocks and 5% return and 6% risk for bonds, 0 correlation and 4% withdrawal

	ATV		ATV		ATV max		return	risk	mdd	omega	mar
SW	\$	197,905.00	\$ 1,174,852.57	0.14%	2.38%	10%	26.61%	4.81	0.09		
Bucket	\$	244,748.07	\$ 3,711,944.47	0.88%	1.58%	16.20%	42.67%	1.81	0.04		
SW/TW	\$	223,627.30	\$ 2,195,026.81	0.95%	1.60%	13.03%	35.17%	1.9	0.05		

Figure 9

#### Case 2: Bad luck at the end of the period-50% bear market in stocks in the last year % fail ATV return risk mdd omega mar max SW \$ 142,471.00 \$ 1,024,537.00 0.22% 0.62% 12% 35.88% 1.53 0.02 Bucket 52,478.61 776,498.00 5.97% -10.70% 21.68% 74.49% Ś Ś 0.02 -0.14 SW/TW Ś 141,390.09 \$ 1,182,058.76 1.23% -0.94% 15.39% 47.56% 0.65 -0.02

Figure 10

#### Case 3: Bad luck at the beginning of the period-50% bear market in stocks in the first year

	ATV		ATV max		return	risk	mdd	omega	mar
SW	\$	101,907.70	\$ 1,085,982.00	2.22%	-3.34%	11.60%	45.87%	0.23	-0.07
Bucket	\$	116,801.71	\$ 1,977,583.65	4.34%	-5.51%	16.11%	54.45%	0.20	-0.10
SW/TW	\$	85,240.70	\$ 1,494,050.00	12%	-14.36	13.89%	61.00%	0.06	-23.54

Figure 11

market occurs in the first year the portfolio is invested (Figure 11). Since the bucket approach is 100% in stocks after the first ten years, we would predict that it would be more adversely impacted by bad luck or a protracted bear market near the end than a systematic withdrawal approach.

As expected, the bucket approach is severely impacted by a bear market in stocks that occurs at the end of the period. The same results would be true to a similar extent if the bear market occurs at any point near the end of an investor's time horizon.

The average terminal value of the bucket method is nearly a third of the value for either SW or SW/TW. Note that average return for the bucket approach across simulations is actually higher than for SW or SW/TW; the expectation is substantially lower due to the presence of adverse outcomes. The % failure rate where an investor runs out of money is an alarming 5.97%, which is more than five times higher than the SW/TW approach. In terms of planning success, the Omega ratio for the bucket approach is substantially lower than both systematic withdrawal methods.

In contrast to the last example, we would expect that "bad luck" at the beginning of the investment period would be more favorable for the bucket method. This is because the stock bucket is shielded from withdrawals for ten years, giving it a chance to recover without having to withdraw too much proportionately at the wrong time. It is arguably even more favorable for the bucket method in real life because the stock market exhibits predictable mean-reversion tendencies. Let's examine the performance on the simulated time series. As expected, the performance of the bucket approach is superior in terms of average terminal value and, relatively speaking, the Omega is roughly on par with SW. This is in contrast to Case 2, where the Omega for SW is much higher than the bucket approach. This implies that SW is hurt much more by initial bad luck, and this is consistent across both SW and SW/TW.

In fact, the SW/TW approach demonstrates the major disadvantage of bad luck at the beginning of the period. The probability of running out of money is nearly three times that of the bucket approach. Furthermore, the Omega for SW/TW is less than a third of the bucket approach.

These results are likely to be true regardless of whether a bear market starts in year one or in the subsequent years that follow—of course, with a lesser magnitude of severity.

The bottom line is that the bucket approach is likely to be superior to a traditional systematic withdrawal approach that is more heavily weighted in equities (like a traditional 60/40 portfolio) when there is "bad luck" early in the investment period. This supports the use of the bucket approach. However, when the "bad luck" comes late, if the bucket approach is invested in traditional asset classes on a buy-and-hold basis, the approach is not as effective.

Let's now look at a case study (Case 4, Figure 12) using some real-life examples to better illustrate the point. In this case we will use actual stock (S&P 500 Index) and bond (10-year Treasurys) data, and run a simulation to compare the bucket approach to the two different systematic withdrawal methods. We use equally sized buckets for the time period and, again, a 50/50 portfolio of stocks and bonds at inception for all strategies.

In this case, the bucket approach falls between both systematic withdrawal

#### Case 4: Actual stock and bond data 1998-present

% run out												
	ATV	max	% fail	return	risk	mdd	omega	mar				
\$	208,290.72	\$ 1,483,782.94	0.35%	2.20%	11.38%	30.30%	3.26	0.07				
\$	172,481.00	\$ 1,536,944.00	1.48%	-0.21%	12.23%	37.97%	0.90	-0.01				
\$	203,850.48	\$ 4,708,047.00	2.78%	-1.06%	14.29%	42.30%	0.73	-0.03				
	\$ \$ \$	\$ 208,290.72 \$ 172,481.00	\$         208,290.72         \$         1,483,782.94           \$         172,481.00         \$         1,536,944.00	ATV         max         % fail           \$ 208,290.72         \$ 1,483,782.94         0.35%           \$ 172,481.00         \$ 1,536,944.00         1.48%	ATV         max         % fail         return           \$         208,290.72         \$         1,483,782.94         0.35%         2.20%           \$         172,481.00         \$         1,536,944.00         1.48%         -0.21%	ATV         max         % fail         return         risk           \$ 208,290.72         \$ 1,483,782.94         0.35%         2.20%         11.38%           \$ 172,481.00         \$ 1,536,944.00         1.48%         -0.21%         12.23%	ATV         max         % fail         return         risk         mdd           \$         208,290.72         \$ 1,483,782.94         0.35%         2.20%         11.38%         30.30%           \$         172,481.00         \$ 1,536,944.00         1.48%         -0.21%         12.23%         37.97%	ATV         max         % fail         return         risk         mdd         omega           \$         208,290.72         \$         1,483,782.94         0.35%         2.20%         11.38%         30.30%         3.26           \$         172,481.00         \$         1,536,944.00         1.48%         -0.21%         12.23%         37.97%         0.90				

Figure 12

methods in terms of Omega and failure rate, but shows lower average terminal values. This is because the correlation between stocks and bonds is highly negative over the time period, and a constantly rebalanced portfolio therefore will have a higher return than the same portfolio mix allowing for drifting allocations. In either case, the bucket approach shows superior planning success metrics than the industrystandard, stock-heavy, time-weighted, static, systematic withdrawal portfolio.

# The Bucket Approach and Active Management

One of the key failures of static asset allocation approaches is that all of them fail to follow trends in both returns and risk. As we have seen, bear markets have been devastating to all forms of static allocation-especially if they happen at the wrong time. This is true regardless of whether one employs a constantly rebalanced approach (nearly impossible to implement) or the bucket approach. The key difference between these two methods is that when buying and holding traditional asset classes, the bucket approach is most affected by bear markets that occur toward the end of the investor's time horizon, while the constantly rebalanced approach (with systematic withdrawal) is most impacted by bear markets that occur at the beginning.

From a practical perspective, since both of these methods were designed for retirement planning, the bucket approach has more psychological appeal. In theory,

people that have just retired are most sensitive to their nest egg and may make rash decisions if they encounter a bear market early on. It is harder for them to be concerned about what may happen if they run out of money at some point in the very distant future. Human nature is to focus on the shorter term. However, the cost of running out of money can be severe-this can mean having no financial options at a point when the client is unlikely to be able to return to work. The bucket approach is more sensitive to this outcome. especially when employing a traditional buy-and-hold approach. Yet as Milevsky stated: bucketing cannot bail you out of a sequence of poor returns.

Active management, in contrast to traditional buy-and-hold investing which penalizes the bucket approach for "bad luck" in the later years, may provide an excellent solution. It focuses on responding to trends in return and volatility by shifting asset allocation throughout the holding period. Most active management approaches that are trend-following based will outperform buy and hold in an extended bear market. As a tradeoff, they may trail on the upside in bull markets. However, in aggregate they produce the smoother return profile that is ideal for financial planning since it typically is not as sensitive to "bad luck." By using active management with the bucket approach, it is possible to produce a nearly ideal scenario that is designed to keep investors invested for the long term while protecting them from events in the future that may devastate their portfolios.

# Combining Fusion with the Bucket Approach

Fusion is our premier active management solution at Flexible Plan. We dynamically combine active management strategies with traditional asset classes into one portfolio to hit a targeted maximum drawdown. The goal is to provide an ideal solution that permits potentially superior returns for a given level of portfolio risk.

Fusion responds to trends in returns and risk and also to the correlations between strategies and assets to dynamically shift portfolio allocations at least monthly. Fusion is the financial equivalent of "cruise control" in your car: as the car starts going too fast above the target speed, the cruise control tells the car to slow down, and if you are going too slow, the cruise control tells the car to speed up.

Fusion has six primary suitability profiles, and for this paper we will focus on the most conservative and the most aggressive (the performance breakdown of the different suitability profiles can be found in the Appendix). Because they have different targeted risk levels, each profile can fill the investment needs of individual buckets.

Using the two most extreme Fusion profiles is comparable to a standard two-bucket approach that uses a very conservative investment in the first bucket and equities in the second bucket for maximum capital appreciation. In the simulation (Figures 13-15) we use the historical performance of the Fusion Indices from 1998 to the present (made available by the New York Stock Exchange on most quote platforms). We use a 4% withdrawal rate, which is consistent with our previous simulations.

In this case, the buckets are divided into two nearly equal-sized portfolios over the 15-year period. In Figure 13 we compare a traditional systematic withdrawal approach that maintains a 50% constantly rebalanced allocation (SW) to both Fusion Conservative and Fusion Aggressive, with a standard two-bucket approach (Bucket).

#### Case 5: Fusion Conservative and Fusion Aggressive Indicesusing the Bucket approach vs SW 1998-present

	ATV		ATV max		return	risk	mdd	omega	mar
SW	\$	1,025,624.00	\$ 5,695,960.77	0	11.60%	11.68%	11.24%	40601.00	1.03
Bucket	\$	1,823,943.57	\$ 18,363,539.92	0	14.24%	16.66%	17.44%	8908.00	0.82
			•	•					

Figure 13



Figure 14

Fusion Bucket	S&P500 and Treasurys Bucket	S&P500 and Treasurys SW	Fusion SW
15.2%	2.3%	3.2%	12.4%
17.2%	18.0%	11.7%	11.9%
30.2%	57.7%	34.5%	19.8%
0.885	0.129	0.277	1.039
1.174	1.043	1.062	1.195
	15.2% 17.2% 30.2% 0.885	15.2%         2.3%           17.2%         18.0%           30.2%         57.7%           0.885         0.129	15.2%         2.3%         3.2%           17.2%         18.0%         11.7%           30.2%         57.7%         34.5%           0.885         0.129         0.277

Figure 15

It is true that consistent with our previous simulations, the Omega is higher for the systematic withdrawal approach versus the bucket approach—thus the probability of financial planning success is higher at first glance. But the average maximum drawdown for the bucket approach was a very tolerable 17.4%—which is less than a third of the maximum drawdown of the S&P 500 over the same time period and only 6% greater than a constantly rebalancing approach. Most investors can tolerate drawdown levels of less than 20%.

Given that an investor's risk and financial objectives are thus being met by either approach, it is worthwhile comparing the difference in returns between the two methods: the bucket approach has a compound return that is nearly 2.5% higher, and an average terminal wealth that is nearly 80% higher than for the systematic withdrawal approach. Fusion is an active management approach that is designed to maximize returns per unit of risk. The cost of using the bucket approach and letting the aggressive portfolio "ride" while starting to withdraw from the conservative bucket is mitigated significantly through risk management.

In the graph (Figure 14), we compare a constantly rebalanced systematic withdrawal approach (SW—50/50 initial allocation, 4% withdrawal) using the Fusion Aggressive and Conservative Index values (purple line) versus the SW approach using the traditional, buy-andhold S&P 500 and Treasurys portfolio (green line). Additionally, we display the two-bucket approach first using traditional assets (red line) and then replacing bonds with Fusion Conservative and stocks with Fusion Aggressive (blue line). The results are summarized in Figure 15. Note that in this case we are able to calculate daily Omega values because we have daily returns available. This makes the difference in values and magnitude much less pronounced than using annual values as we did in the previous simulations.

What is striking is that the bucket approach outperforms the systematic withdrawal approach using active management (Fusion), but underperforms when using the passive approach with stocks and bonds as is typically employed in the financial industry. The daily Omega values are nearly identical for Fusion SW and Fusion with the bucket approach, but the returns are nearly 3% higher for the Fusion/bucket approach. In contrast, the returns are nearly 1% lower for the bucket approach with the passive portfolios, and the Omega was also slightly lower.

Whether using active or passive management, the drawdowns for the bucket approach are higher than for the systematic withdrawal method, but the Fusion actively managed bucket approach has a more tolerable 30% drawdown versus the nearly 60% drawdown with the passive portfolio. History provides a good example of how the bucket approach can survive being unlucky, even if faced with a bear market near the end of the period. Investors in a passive portfolio would have been exposed 100% to stocks when the 2008 bear market began, which is why the resulting drawdown was so severely felt by them. Few investors would be able to sustain that level of drawdown (almost 60%) and still continue with the same asset allocation. Fusion experienced a maximum drawdown of about half that level.



# Which Fusion Profile for which Bucket?

We performed 25,000 simulations to determine which Fusion suitability profiles were best for different levels of targeted returns (withdrawal rates). We compared these profiles to stocks (S&P 500), bonds (Treasurys), and a 60/40 portfolio of stocks and bonds. In this case we used a standard systematic withdrawal method since there was only one suitability profile used in each of the groups in the simulations. The purpose was to determine which Fusion suitability index would be ideal for a bucket with a given target return.

Figure 16 illustrates that by simply using Fusion Enhanced Income and Balanced one can achieve the highest probability of success (bolded results).

Figure 17 shows the greatest MAR (ratio of return to max drawdown) can be achieved by the same profiles.

Figure 18 adds time horizon into the index to discover the probability of success for Fusion and traditional asset classes in meeting both the withdrawal needs and the requisite time horizon. All of the Fusion profile indexes demonstrate a better than 90% probability of success. In contrast, the buy-and-hold asset classes and 60/40 portfolio seriously lag.

Summing up our findings, what is striking about the results deployed in Figures 16-18 is that all six Fusion profiles outperform the asset classes and a 60/40 portfolio at all levels of return/withdrawal rates. As expected, as the target return increases, the optimal Fusion profile also increases its risk profile. However, since the maximum target returns are modest in relation to historical Fusion Index returns, Fusion Balanced is the most aggressive portfolio required to maximize the probability of planning success at the 8% target return level. In addition, it's clear that rather than limiting one's investments to both the more limited risk and return of the Moderate portfolio, the use of even more aggressive Fusion portfolios (designed for higher returns) can be justified and still potentially outperform the use of traditional buy-and-hold asset categories using the bucket system.

#### A Simulation of Different Fusion Suitability Indexes versus Standard Asset Classes and Portfolios at Different Target Returns/ Withdrawal Rates using the Omega Statistic (using 1998-2013 data)

	3%	4%	5%	6%	7%	8%
Fusion Conservative	379.0	102.0	36.2	12.6	4.4	1.7
Fusion Moderate	523.0	249.8	109.0	56.8	29.7	14.4
Fusion Enhanced Income	530.0	296.0	134.0	68.0	37.4	21.5
Fusion Balanced	407.0	228.0	121.0	81.9	46.8	26.3
Fusion Growth	211.6	133.0	82.4	55.9	37.3	25.5
Fusion Aggressive	98.0	57.8	45.0	30.7	22.6	16.7
S&P 500	1.3	1.0	0.8	0.6	0.4	0.3
Treasurys (10-year)	11.1	4.7	2.0	0.8	0.4	0.2
60/40 S&P 500 and Treasurys	4.6	2.8	1.7	1.0	0.6	0.4

Figure 16

A Simulation of Different Fusion Profiles versus Standard Asset Classes and Portfolios at Different Target Returns/Withdrawal Rates using the Average MAR (Return/Maximum Drawdown) Statistic (using 1998-2013 data)

3%	4%	5%	6%	7%	8%
2.85	1.74	1.06	0.58	0.27	0.07
3.43	2.61	1.95	1.44	1.04	0.70
3.43	2.69	2.10	1.63	1.24	0.92
3.10	2.55	2.05	1.72	1.36	1.07
2.35	2.06	1.77	1.48	1.27	1.06
1.75	1.50	1.33	1.14	1.00	0.89
0.04	0.00	-0.04	-0.07	-0.11	-0.14
0.50	0.27	0.10	-0.02	-0.10	-0.15
0.27	0.16	0.08	0.00	-0.05	-0.11
	2.85 3.43 3.43 3.10 2.35 1.75 0.04 0.50	2.85         1.74           3.43         2.61           3.43         2.69           3.10         2.55           2.35         2.06           1.75         1.50           0.04         0.00           0.50         0.27	2.85         1.74         1.06           3.43         2.61         1.95           3.43         2.69         2.10           3.10         2.55         2.05           2.35         2.06         1.77           1.75         1.50         1.33           0.04         0.00         -0.04           0.50         0.27         0.10	2.85         1.74         1.06         0.58           3.43         2.61         1.95         1.44           3.43         2.69         2.10         1.63           3.10         2.55         2.05         1.72           2.35         2.06         1.77         1.48           1.75         1.50         1.33         1.14           0.04         0.00         -0.04         -0.07           0.50         0.27         0.10         -0.02	2.85         1.74         1.06         0.58         0.27           3.43         2.61         1.95         1.44         1.04           3.43         2.69         2.10         1.63         1.24           3.10         2.55         2.05         1.72         1.36           2.35         2.06         1.77         1.48         1.27           1.75         1.50         1.33         1.14         1.00           0.04         0.00         -0.04         -0.07         -0.11           0.50         0.27         0.10         -0.02         -0.10

Figure 17

#### A Simulation of Different Fusion Profiles versus Standard Asset **Classes and Portfolios at Different Target Returns/Withdrawal Rates** using Probability of Success over 5 Years (using 1998-2013 data)

5-Year Horizon	3%	4%	5%	6%	7%	8%
Fusion Conservative	98.72%	96.75%	92.75%	85.76%	75.23%	61.55%
Fusion Moderate	99.16%	98.43%	97.20%	95.27%	92.40%	88.35%
Fusion Enhanced Income	99.11%	98.45%	97.41%	95.83%	93.56%	90.41%
Fusion Balanced	99.00%	98.44%	97.61%	96.44%	94.84%	92.72%
Fusion Growth	98.46%	97.84%	97.01%	95.94%	94.58%	92.87%
Fusion Aggressive	97.33%	96.51%	95.50%	94.26%	92.77%	91.01%
S&P 500	62.90%	58.74%	54.48%	50.18%	45.86%	41.60%
Treasuries (10-Year)	83.71%	74.17%	62.36%	49.24%	36.21%	24.61%
60/40 S&P and Treasurys	76.14%	69.85%	62.91%	55.52%	47.93%	40.42%
20-Year Horizon	3%	4%	5%	6%	7%	8%
Fusion Conservative	100.00%	99.99%	99.82%	98.38%	91.36%	72.16%
Fusion Moderate	100.00%	100.00%	99.99%	99.96%	99.79%	99.15%
Fusion Enhanced Income	100.00%	100.00%	99.99%	99.97%	99.88%	99.55%
Fusion Balanced	100.00%	100.00%	100.00%	99.98%	99.94%	99.82%
Fusion Growth	100.00%	100.00%	99.99%	99.98%	99.93%	99.83%
Fusion Aggressive	99.99%	99.99%	99.97%	99.92%	99.82%	99.63%
S&P 500	74.48%	67.06%	58.91%	50.35%	41.77%	33.57%
Treasurys (10-Year)	97.53%	90.28%	73.55%	48.48%	24.01%	8.48%
	92.25%	85.09%	74.51%	60.94%	45.87%	31.39%

Figure 18

### Conclusion

In this paper we compared the bucket approach with a more traditional systematic withdrawal approach with a constantly rebalanced portfolio. We show through simulation that the bucket approach is a tradeoff that can increase returns versus a traditional approach and tends to reduce the impact of bad luck at the beginning of the investment period, but it is more sensitive to the impact of bad luck at the end of the planning period. In general, a traditional approach tends to have a higher probability of planning success because diversification is maximized and risk is minimized by rebalancing to a constant mix.

However, we discussed the possibility of using active management within the

bucket approach—which has major implications at the aggressive end of the portfolio. Our studies show that this reduces the gap in planning success and downside risk between using either the bucket approach or the systematic withdrawal method to the point where there is a negligible difference between the two in practical terms. At the same time, however, the bucket method with active management shows the potential for much higher returns, and given its greater psychological appeal to investors, it is a more ideal combination.

Financial planning and investment management cannot be applied in isolation. Both need to be comprehensively integrated in order to achieve client objectives. Dynamic allocation across investment strategies and asset classes along an efficient frontier represents a theoretically desirable approach to optimize investment performance for a targeted level of return.

In our simulations, active management outperformed whether using the traditional systematic method or the more psychologically appealing bucket approach. Furthermore, in implementing the bucket approach we found that using different Fusion suitability profiles for different target return buckets proved to be a promising method of further integration to maximize the outcome from a bucket approach to financial planning.

# Bucket Investing vs. Traditional Portfolio Management

#### Bucket investing

- Multiple portfolios
- · Each with a different time horizon
- · Matches assets to future liabilities
- Always reallocates to the shorter-term buckets
- Always withdraws from the shortestterm bucket

# Systematic withdrawal from a single portfolio

- Single portfolio
- Meant to last the longest investment time horizon
- Single investment policy
- · Needs to be very diversified
- · Needs to constantly rebalance
- Withdrawals taken from single portfolio on a systematic basis

### Bucket Investing Disadvantages

- The myth of time diversification
- Wide range of outcomes possible
- Time and discipline needed for reallocation
- Cost of inefficiencies
- Timing of losses

### Bucket Investing Advantages

- Moves in tune with investor psychology
- Takes advantage of "mental accounting"
- Each portfolio customized for a specific time horizon
- Focuses on survival of principle
- Lets long-term investments compound longer

#### Appendix

# What is Bucket Investing and how does it work?

The example (Figure a) shows a 3-bucket system with a 5% systematic withdrawal each year. Bucket 1 has a 5-year lifecycle while Bucket 2 has a 10-year lifecycle. Bucket 3 remains open until its funds are depleted. The systematic withdrawal occurs only in the bucket that is currently "active." After the 5th year, bucket 1 will transfer any remaining money into bucket 2, which will then begin experiencing yearly withdrawals. If any bucket's money is depleted before the maximum holding period for that bucket has elapsed, the bucket system will transfer money out of the current bucket at the end of the year into the next bucket, which will then receive the annual withdrawals. This systematic

withdrawal algorithm applies to all of the buckets in the system except for the final bucket, which will remain open in perpetuity or until depleted.

To build simulations of potential portfolio outcomes, we generated 10,000+ bucket system portfolios using a simulation application designed specifically to model the bucket system. We used the algorithms outlined in Figure b & Figure c to generate each portfolio's yearly returns. Each equity path represents the portfolio's year-end price streams. (Figure a shows one simulated equity path using end-ofyear prices.)

# Monte Carlo Simulation

To illustrate this concept, we performed a series of Monte Carlo simulations, which is a widely accepted practice for modelling financial time series within the industry. Monte Carlo simulation uses what is known as risk-neutral valuation to generate return simulations. We used stochastic differential equations with geometric Brownian motion to generate random time series based on a given mean and standard deviation.

			Ra	te of Retu	rns			Bucket 1	Bucket 2	Bucket 3	Year End
% W/D	\$ W/D	Year	Bucket 1	Bucket 2	Bucket 3		Start	30000	30000	30000	90000
5%	4500	Year 1	4.40%	7.43%	-0.80%		Year 1	26820	32228.57	29760	88808.57
5%	4500	Year 2	-7.20%	4.86%	51.60%	Buc	ket 1 Year 2	20388.96	33793.96	45116.16	99299.08
5%	4500	Year 3	3.20%	-4.57%	18.76%	Hol	lding Year 3	16541.41	32249.09	53579.95	102370.5
5%	4500	Year 4	-0.40%	9.43%	12.80%	Per	riod Year 4	11975.24	35289.72	60438.19	107703.1
5%	4500	Year 5	-0.60%	4.00%	-11.20%		Year 5	7403.39	36701.31	53669.11	97773.81
5%	4500	Year 6		-9.43%	28.40%		Year 6		35446.26	68911.14	104357.4
5%	4500	Year 7		5.14%	12.80%	Buc	ket 2 Year 7		32769.21	77731.76	110501
5%	4500	Year 8		-0.57%	2.80%	Hol	Iding Year 8		28081.95	79908.25	107990.2
5%	4500	Year 9		4.86%	3.20%	Per	riod Year 9		24945.93	82465.31	107411.2
5%	4500	Year 10		11.43%	-10.80%		Year 10		23296.9	73559.06	96855.96
5%	4500	Year 11			29.00%		Year 11			81895.52	81895.52
5%	4500	Year 12			-16.00%	Buc	ket 3 Year 12			101145.2	101145.2
5%	4500	Year 13			52.80%		The Year 13			80461.98	80461.98
5%	4500	Year 14			6.40%		Year 14			118445.9	118445.9
5%	4500	Year 15			-10.00%		cket Year 15			121526.4	121526.4
5%	4500	Year 16			36.40%		d No Year 16			104873.8	104873.8
5%	4500	Year 17			23.00%		Iding Year 17			138547.9	138547.9
5%	4500	Year 18			34.00%		riod Year 18			165913.9	165913.9
5%	4500	Year 19			-14.00%		Year 19			217824.6	217824.6
5%	4500	Year 20			5.20%		Year 20			182829.1	182829.1

Figure a

$$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t$$

Figure b

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right)$$

Figure c

$$Z_0 = R\cos(\Theta) = \sqrt{-2\ln U_1}\cos(2\pi U_2)$$
$$Z_1 = R\sin(\Theta) = \sqrt{-2\ln U_1}\sin(2\pi U_2)$$

Figure d

A stochastic process (St) is calculated using a percentage drift (Mu), percentage volatility (Sigma) and Brownian motion factor (dWt). The variable, "dt," is the time step used for Mu and Sigma values (Figure b). The analytical solution for Figure b is Figure c.

The Box-Muller transformation (Figure d) is used to generate normally distributed random numbers using independent random numbers:  $U_1$  and  $U_2$  are independent random numbers that are uniformly distributed between 0 and 1. We then use the Mersenne Twister, a high-quality, pseudo-random number

generation algorithm to generate these independent, random numbers.

This gives you two independent random numbers,  $Z_0$  and  $Z_1$ , each with a standard normal distribution.

To generate a full time series, we recursively call the above algorithm for the required number of data points. We used yearly CAGR and standard deviation for Mu and Sigma where the time step is one year and the initial portfolio value is 100,000 for all simulations.

# **Bucket Return Correlation**

In the real world, single bucket returns have some degree of correlation to the other buckets. To simulate this important feature, we used the Cholesky Decomposition, or Cholesky Factorization algorithm, to integrate correlations into our individual simulations, for example, using the equations in Figure e to generate random returns for two correlated assets.

 $X_1$  and  $X_2$  are normally distributed, correlated random numbers, while  $Z_1$  and  $Z_2$  are normally distributed, uncorrelated random numbers. In applying the LU Decomposition algorithm to the correlation matrix to an uncorrelated sample, we can generate randomly correlated financial time series for the buckets.

# Omega Values for Probability of Success

To calculate Omega on these portfolios, we calculate the CAGR (Compound Annual Growth Rate or Return) for each simulation using year-end prices for each bucket system portfolio.

The formulae of Omega (Figure f where F(x) is the probability density function of return x) is implemented as OMEGA = Excess Sum of All CAGR Values above Threshold / Deficiency Sum of All CAGR Values bellow Threshold Geometric OMEGA, or GOMEGA = Excess Average of All CAGR Values above Threshold / Deficiency Average of All CAGR Values bellow Threshold

We also used a yearly % ranking of all simulations (year-end price series) to generate the 20th, 50th, and 80th percentile of equity paths.

Normal Investment Simulation vs. Bucket Investment Simulation for 20-Year Period (Monte Carlo Simulations):

- 0.5 Correlation between all buckets
- 5% Systematic Withdrawal
- 4% Threshold for OMEGA
- 10% CAGR and 10% Risk for all buckets
- \$100,000 Initial investment equallyweighted to each bucket
- 10,000 Simulations for each system

$$x_1=z_1$$
 And  $x_2=
ho z_1+\sqrt{1-
ho^2}z_2$ 

Figure e

$$\Omega(r) = \frac{\int_r^\infty (1 - F(x)) \, dx}{\int_{-\infty}^r F(x) \, dx}$$

Figure f

System	Average CAGR	Average Risk	OMEGA	Running Out Money
Normal	6.20 %	9.78 %	5.75	2.80 %
2 Buckets	6.34 %	9.50 %	7.55	2.02 %
3 Buckets	6.45 %	9.11 %	8.99	1.47 %
4 Buckets	6.47 %	8.61 %	11.03	0.96 %
5 Buckets	6.47 %	8.05 %	13.74	0.68 %



# Running Out Money - % of All Simulation



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Indexes do not predict or project the performance of an investment or investment strategy as one cannot invest directly in an index. While FPI's Fusion strategies are based on the Fusion Indexes, and the FPI Self-adjusting Trend Following ("STF") strategy is based on the STF Index, the Indexes reflect the theoretical performance an investor would have obtained had it invested in the manner shown and does not represent returns an investor actually attained, as investors cannot invest directly in an index. No representation is being made that any client will or is likely to achieve results similar to those presented.

The Fusion Indexes are constructed illustrating the historical allocated performance from various sources, including actual returns of FPI's model account strategies, ETFs and open-end mutual funds (including long/short and/or leveraged funds). From January 1998 to December 2013, the Indexes' performance was calculated by FPI. Index calculations from January 2014 forward were performed by the NYSE Group, Inc. From January 1998 to December 2013, there have been no assets managed under the Fusion Index rules at FPI. The Fusion mathematical algorithm has been in use by FPI since February 2013. The Self-adjusting Trend Following Index rules have been used to manage assets at FPI since July 2009. FPI reserves the right to make enhancements to the Indexes' methodologies.

The Fusion Indexes and the STF Index contain no management or advisory fees, other than the internal fees and expenses reflected in the NAV of the mutual funds or ETFs used. A client account in an FPI-offered Fusion or STF strategy will incur advisory fees; additional fees may apply including transactions and trading costs determined by the Custodian of the account. These fees and costs will decrease the return experienced by a client. Individual client account results will vary from the Indexes' returns. Current and prospective clients should not assume that future performance will be the same or profitable. Distributions have been reinvested. When provided, dividends are reinvested for indexes. In those cases where indexes do not provide dividend information, those returns would be understated. As individual tax rates vary, taxes have not been considered.

Performance for the period, generally encompassing 1999 and the first quarter of 2000 was driven by substantial price appreciation in a small number of equity issues, notably in technology sectors, traded primarily on the NASDAQ. Such performance is historical information and should not be relied upon as representative of investment performance of any strategy to the current date nor be extrapolated into expectations for the future. Inquiry for current results is advised, in light of the adverse market performance of many indices commencing in 2000.